UNIT- II

PROBLEMS

(1) When a card is drawn from a pack of cards Find the probability of getting

(a) a diamond card
(b) a King card
(c) a red card
(d) a club card
(e) a face card
(f) a queen card
(g) a '10' numbered card
(h) a black king
(i) a red face card
(j) an ace card
(k) a hearts card
(l) a spade king

Sol:

Total no. of cards = 52

(a) no. of diamond cards (D) = 13

$\therefore P(D) = \frac{13}{52}$

(b) no. of King cards (K) = 4

$P(K) = \frac{4}{52}$

(c) no. of red cards (R) = 26

(13 hearts + 13 diamonds)

$\therefore P(R) = \frac{26}{52}$

(d) no of club cards (C) = 13

$$P(C) = \frac{13}{52}$$

(e) no. of face cards (F) = 16

(4-kings + 4-queens + 4-ace + 4 jacks)

$$P(F) = \frac{16}{52}$$

(f) no of Queen cards (Q) = 4

$$P(Q) = \frac{4}{52}$$

(g) no of red face cards = 8

$$\therefore P(RF) = \frac{8}{52}$$

(h) no of ace cards = 4

$$P(A) = \frac{4}{52}$$

(i) no of hearts cards = 13

$$P(H) = \frac{13}{52}$$

(j) no of spades = 13

$$P(S) = \frac{13}{52}$$

(2) A bag contains 3 red, 5 green and 7 black balls. A ball is drawn at random. Find the

(i) probability of getting a red ball

Sol Total no. of balls = 3+5+7 = 15

no. of red balls = 3

$$P(R) = \frac{3}{15}$$

(ii) Prob of getting green ball

Sol no. of green balls = 5

$$P(G) = \frac{5}{15}$$

(iii) Prob of getting red or green ball

Sol: no. of red balls = 3

no. of green balls = 5

R+G = 3+5 = 8

∴ Prob of getting red or green = $\frac{8}{15}$

(iv) Prob of getting red or black balls

Sol: no. of red balls = 3

no. of black balls = 7

R+B = 3+7 = 10

Prob of getting red or black ball = $\frac{10}{15}$

(3) Two cubical dice are thrown. Find the probability of

(i) getting the sum of the numbers on their faces is 8.

Sol: Total no. of outcomes $6 \times 6 = 36$ (b)

To get sum of the numbers is 8

$(2,6), (3,5), (4,4) (5,3) (6,2)$

$\therefore n = 5$

$P(E) = \frac{5}{8}$

(ii) Sum of the numbers is 10

Sol: $n = 36$

to get Sum 10 $\Rightarrow$ $(4,6) (5,5) (6,4)$

$n = 3$

$P(E) = \frac{3}{36}$

(iii) to get a sum of 10 or 11

Sol: $n = 36$

to get 10 $\Rightarrow$ $(4,6) (5,5) (6,4)$

to get 11 $\Rightarrow$ $(5,6) (6,5)$

$n = 3+2 = 5$

$P(E) = 5/36$

(iv) of getting a sum less than 4.

Sol: $n = 36$ to get Less than 4

ie $3 \rightarrow (1,2) (2,1)$

$2 \rightarrow (1,1)$

$n = 2+1 = 3$ $P(E) = 3/36$

(v) of getting Sum atleast 10

Sol: $n = 36$

To get atleast 10 means getting 10, 11, or 12.

for 10 → (4,6) (5,5), (6,4)
11 → (5,6) (6,5)
12 → (6,6)

$m = 3+2+1 = 6$

$$P(E) = 6/36 = 1/6$$

(vi) of getting a sum of 2 or 8? (NOV 2001)

Sol: $n = 36$

to get a sum 2 = (1,1)
a sum 8 = (2,6), (3,5) (4,4) (5,3) (6,2)

$m = 6$

$$P(E) = \frac{6}{36} = \frac{1}{6}$$

(vii) of getting a prime sum

Sol: $n = 36$

to get a prime sum

ie 2 → (1,1)
3 → (1,2) (2,1)
5 → (1,4) (2,3) (3,2) (4,1)
7 → (1,6) (2,5) (3,4) (4,3) (5,2) (6,1)
11 → (5,6) (6,5)

$m = 15$

$$P(E) = \frac{15}{36}$$

(vii) To get a odd sum

Sol: $n = 36$

To get odd sum → (1,2) (1,4) (1,6)
(2,1) (2,3) (2,5)
⋮
(6,1) (6,3) (6,5) } 18

$n = 18$

$$P(E) = \frac{18}{36} = \frac{1}{2}$$

(4) A Card is drawn at random from a well shuffled pack of playing cards (52 in number). Find the probability that it is either a spade or a diamond

APRIL 2003

Sol:

n.o of Cards = 52

n.o of Spades (S) = 13

n.o of diamonds (D) = 13

Spades and diamonds are mutually exclusive.

$S+D = 13+13 = 26$

∴ Prob of getting a spades or diamonds Card is $P(E) = \frac{26}{52} = 1/2$

OTHER METHOD

$P(S) = \frac{13}{52}$ $P(D) = \frac{13}{52}$ $P(S \cup D) = ?$ $P(S \cap D) = 0$

$P(S \cup D) = P(S) + P(D) - P(S \cap D)$

$= 13/52 + 13/52 - 0 = \frac{26}{52} = 1/2$

(5) Determine the probabilities of the following events in drawing a card at random from a standard deck of 52 cards

(a) A seven (b) A black (c) An ace or a king

(d) A black two or a black three

(e) A red face card NOV 2000.

Sol:

Total no of card = 52

(a) no of 'seven' numbered cards = 4

∴ Prob of getting a seven is

$$P(E) = \frac{4}{52}$$

(b) no of black cards = (13+13) = 26

(13 - clubs + 13 spades)

∴ Probability of Black Card is

$$P(B) = \frac{26}{52}$$

(c) no of Ace cards = 4

no of King cards = 4

A+K = 4+4 = 8

Ace and Kings are Mutually Exclusive

Prob of drawing an Ace or a King is

$$P(E) = \frac{8}{52}$$

(d) no of black '2' numbered cards = 2
[club '2' + spade '2']

no of black '3' numbered cards = 2
[club '3' + spade '2']

Probability of drawing a black two or a black three is $P(E) = \frac{2+2}{52} = \frac{4}{52}$

(e) no of red face cards = 4+4=8

Red → { Diamond → (K+Q+J+A), Hearts → (K+Q+J+A) } → 4+4=8

∴ Probability of drawing a red face card is

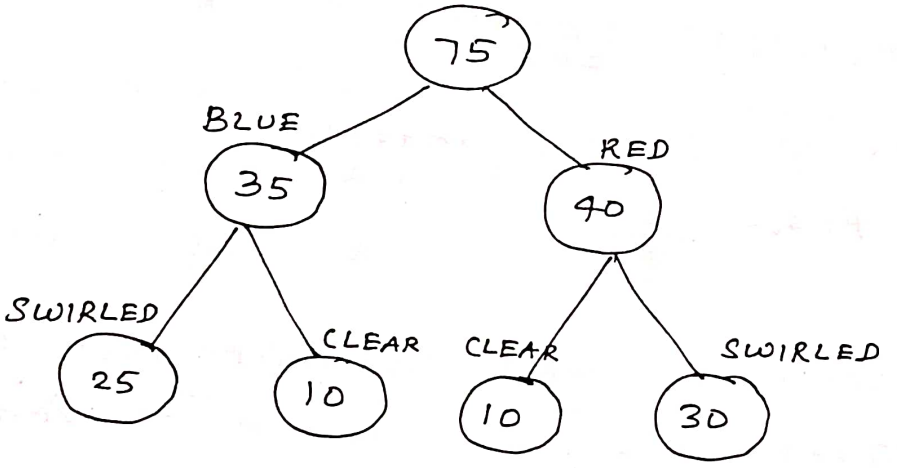
$P(E) = \frac{8}{52}$

(6) An urn contains 75 marbles, 35 are blue and 25 of these blue marbles are swirled. The rest of them are red and 30 of the red ones are swirled. The marbles that are not swirled are clear.

What is the probability of drawing

(a) a blue marble from the urn?
(b) a clear marble from the urn?
(c) a blue swirled marble?
(d) a red, clear marble?
(e) a swirled marble?

Sol: Total marbles = 75 (9)



no. of Blue marbles (B) = 35

no. of Red marbles (R) = 40

no. of Swirled Marbles (S) = 25+30 = 55

no. of clear marbles (C) = 10+10 = 20

no. of Blue and Swirled marbles ($B \cap S$) = 25

no. of Blue and clear marbles ($B \cap C$) = 10

no. of Red and Swirled marbles ($R \cap S$) = 30

no. of Red and Clear marbles ($R \cap C$) = 10

(a) Prob of blue marble

$$P(B) = \frac{35}{75}$$

(b) Prob of Clear marble

$$P(C) = \frac{20}{75}$$

(c) Prob of a blue Swirled marble

$$P(B \cap S) = \frac{25}{75}$$

(d) Prob of red, clear marble

$$P(R \cap C) = \frac{10}{75}$$

(e) Prob of swirled marble

$$P(S) = \frac{55}{75}$$

(7) A bag Contains 7 red, 12 white and 4 green balls. Three balls are drawn at random what is the probability that

(i) 3 balls drawn are all white

(ii) 3 balls drawn are one of each colour.

Sol:

Total no of balls $7+12+4 = 23$

Three balls are drawn at random

∴ Total no. of ways of drawing 3 balls from 23 balls is $C(23,3)$ ways

$$n = C(23, 3)$$

(i) prob of drawing 3 white balls

no. of white balls $(w) = 12$

Three white balls from 23 white balls can be drawn in $C(12, 3)$ ways.

$$\therefore m = C(12, 3)$$

$$P(E) = \frac{C(12,3)}{C(23,3)}$$

(ii) 3 balls drawn are one of each colour (ie) 1 red, 1 white and 1 green ball.

1 red from 7 red can be obtained in $C(7,1)$ ways
and 1 white from 12 white " in $C(12,1)$ ways
and 1 green from 4 green " in $C(4,1)$ ways.

$$P(E) = \frac{C(7,1) \cdot C(12,1) \cdot C(4,1)}{C(23,3)}$$

$$P(E) = \frac{7 \cdot 12 \cdot 4}{C(23,3)} = \frac{336}{1771}.$$

(8) Three cards are drawn from a pack of cards. Find

(i) The probability of getting 3 club cards

Sol Total no of cards = 52

3 cards drawn at random.

3 cards from 52 cards can be drawn in $C(52,3)$ ways

$$n = C(52,3)$$

no of club cards = 13

3 club cards from 13 club cards can be obtained in $C(13,3)$ ways

$$m = C(13,3)$$

$$P(E) = \frac{C(13,3)}{C(52,3)}$$

(ii) The probability of getting 3 King cards

Sol: no of king cards = 4

$$n = C(4,3)$$

$$P(E) = \frac{C(4,3)}{C(52,3)}$$

(iii) The prob of getting 3 face cards

Sol: no. of face cards = 16

$$m = C(16,3)$$

$$P(E) = \frac{C(16,3)}{C(52,3)}$$

(iv) The prob of getting 3 red cards

Sol: no. of red cards = 26

$$m = C(26,3)$$

$$P(E) = \frac{C(26,3)}{C(52,3)}$$

(v) The prob of getting 2 diamonds and 1 spades

Sol: no. of diamonds = 13

no. of spades = 13

$$m = C(13,2)\,C(13,1)$$

$$P(E) = \frac{C(13,2)\,C(13,1)}{C(52,3)}$$

(vi) The prob of getting 3 hearts or 3 diamonds

Sol: no. of hearts cards = 13

no of diamond cards = 13

$$m = C(13,3) + C(13,3)$$

$$P(E) = \frac{C(13,3) + C(13,3)}{C(52,3)} = \frac{2 \cdot C(13,3)}{C(52,3)}$$

(vii) The prob of getting 3 Kings or 3 Queens

Sol: no. of King Cards = 4

no. of Queen Cards = 4

$$m = c(4,3) + c(4,3)$$

$$P(E) = \frac{c(4,3) + c(4,3)}{c(52,3)}$$

$$= \frac{2 \cdot c(4,3)}{c(52,3)}$$

(viii) The prob of getting 2 Kings and 1 Queen

Sol: no. of Kings = 4

no. of Queens = 4

$$m = c(4,2) \cdot c(4,1)$$

$$P(E) = \frac{c(4,2) \cdot c(4,1)}{c(52,3)}$$

(ix) The prob of getting 1 King, 1 Queen & 1 Ace

Sol: no. Kings = no. of Queens = no. of Ace = 4.

$$m = c(4,1)\, c(4,1)\, c(4,1)$$

$$P(E) = \frac{c(4,1)\, c(4,1)\, c(4,1)}{c(52,3)}$$

(x) The prob of getting 1 hearts, 1 club & 1 diamond

Sol. no. of hearts = no. of clubs = no. of diamonds = 13

$$m = c(13,1)\, c(13,1)\, c(13,1)$$

$$P(E) = \frac{c(13,1)\, c(13,1) \cdot c(13,1)}{c(52,3)}$$

(9) A bag contains 5 white, 4 green and 6 yellow coloured balls. Three balls are drawn at random. Find

(a) The probability of getting 3 white balls

Sol: Total no of balls $5W + 4G + 6Y = 15$

3 balls are drawn at random

Total no of ways of drawing 3 balls from 15 balls $= C(15,3)$

$n = C(15,3)$

no of white balls $= 5$

$\therefore m = C(5,3)$

$$P(E) = \frac{C(5,3)}{C(15,3)}$$

(b) The probability of getting 3 green balls

Sol: no of green balls $= 4$

$m = C(4,3)$

$$P(E) = \frac{C(4,3)}{C(15,3)}$$

(c) The probability of getting 2 white and 1 green balls.

Sol: no of white balls $= 5$

no of green balls $= 4$

$m = C(5,2) \cdot C(4,1)$

$$P(E) = \frac{C(5,2)\,C(4,1)}{C(15,3)}$$

(d) The probability of getting 1 white and 2 yellow balls

Sol: no. of white balls = 5

no. of yellow balls = 6

$$m = c(5,1)\, c(6,2)$$

$$P(E) = \frac{c(5,1)\, c(6,2)}{c(15,3)}$$

(e) The prob of getting 3 white or 3 green balls

Sol: no. of white balls = 5

no. of green balls = 4

$$m = c(5,3) + c(4,3)$$

$$P(E) = \frac{c(5,3) + c(4,3)}{c(15,3)}$$

(f). The probability of drawing 3 different Coloured balls

Sol: Three different Coloured balls mean 1 white, 1 green and 1 yellow

$$m = c(5,1) \cdot c(4,1)\, c(6,1)$$

$$= 5 \times 4 \times 6$$

$$P(E) = \frac{5 \times 4 \times 6}{c(15,3)}$$

Problems on Addition Theorem

(1) A card is drawn from a pack of cards. Find the probability of getting a diamond or a king card?

Sol: Total no. of cards = 52

no of king cards (K) = 4

no of Diamond cards (D) = 13

$\therefore P(D) = \frac{13}{52}$ $P(K) = \frac{4}{52}$

There exists only one card which is both belongs to diamond and king.

$D \cap K = 1$

$P(D \cap K) = 1/52$

$P(D \cup K) = ?$

$$P(D \cup K) = P(D) + P(K) - P(D \cap K)$$

$$= \frac{13}{52} + \frac{4}{52} - \frac{1}{52}$$

$$= \frac{13+4-1}{52} = \frac{16}{52}$$

(2) A card is drawn from a pack of Cards. Find the probability of getting a Diamond or a club card.

Sol: Total no. of cards = 52

no of Diamond cards (D) = 13

no of club cards (C) = 13

$P(D) = \frac{13}{52}$ $P(C) = \frac{13}{52}$

Diamonds and clubs are mutually exclusive

$\therefore D \cap C = \phi$ $P(D \cap C) = 0$

$P(D \cup C) = ?$

في مرحلة الإنتاج pipeline

$P(D \cup C) = P(D) + P(C)$

$= \frac{13}{52} + \frac{13}{52} = \frac{26}{52} = \frac{1}{2}$

(3) An inspector of a pipeline company has the task of comparing the reliability of two pumping stations. Each station is susceptible to two kinds of failure; pump failure and Leakage (تسرب). When either or both occur, the station must be shut down. The failure probabilities of both stations are given below.

سرقة التأثير / susceptible

Station	P(Pump failure)	P(Leakage)	P(Both)
A	0.07	0.10	0
B	0.09	0.12	0.06

Which station has the higher probability of being shut down? (APRIL 2007)

Sol: Let F = pump failure
L = Leakage

Given either or both occurs the station must be shut down.

For station A

$P(F) = 0.07$

$P(L) = 0.10$

$P(F \cap L) = 0$

$P(F \cup L) = ?$

$$P(F \cup L) = P(F) + P(L) - P(F \cap L)$$
$$= 0.07 + 0.10 - 0$$
$$= 0.17$$

For station B

$P(F) = 0.09$

$P(L) = 0.12$

$P(F \cap L) = 0.06$

$P(F \cup L) = ?$

$$P(F \cup L) = P(F) + P(L) - P(F \cap L)$$
$$= 0.09 + 0.12 - 0.06$$
$$= 0.15$$

Since $P(F \cup L)$ for station A is 0.17.

∴ Station A has higher probability of being shut down.

(4) A construction Company is bidding for two contracts, A and B. The probability that the Company will get Contract A is 3/5. The probability that the Company will get Contract B is 1/3, and the probability that the Company will get both the

Contracts is 1/8. what is the probability that the Company will get Contract A or B?

Sol: Given $P(A) = 3/5$

$P(B) = 1/3$ $\qquad P(A \cup B) = ?$

$P(A \cap B) = 1/8$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= 3/5 + 1/3 - \frac{1}{8}$$

$$= \frac{3(24) + 1(40) - 1(15)}{120}$$

$$= \frac{72 + 40 - 15}{120} = \frac{97}{120}$$

(5) The probability that a Contractor will get a plumbing Contract is 2/3, and the probability that he will not get an electric Contract is 5/9. If the probability of getting atleast one Contract is 4/5, what is the probability that he will get both?

Sol: Let B = plumbing Contract

E = electric Contract

Given $P(B) = 2/3$

$P(\bar{E}) = 5/9$

$P(B \cup E) = 4/5$ $\qquad P(B \cap E) = ?$

$P(\bar{E}) = 5/9 \Rightarrow P(E) = 1 - P(\bar{E})$

$= 1 - 5/9$

$= \frac{4}{9}$.

$P(B \cup E) = P(B) + P(E) - P(B \cap E)$

$P(B \cap E) = P(B) + P(E) - P(B \cup E)$

$= \frac{2}{3} + \frac{4}{9} - \frac{4}{5}$

$= \frac{2(15) + 4(5) - 4(9)}{45}$

$= \frac{30 + 20 - 36}{45} = \frac{14}{45}$

$P(B \cap E) = \frac{14}{45}$

(6) A card is drawn from cards numbered 1 to 50. If a card is drawn from a pack of cards, find the probability of getting a card which is a multiple of 3 or multiple of 5.

Sol: Total no. of cards = 50

Let E_1 = multiple of 3

{3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39, 42, 45, 48}

no. of multiples of 3 = (16)

$\therefore P(E_1) = \frac{16}{50}$

Let E_2 = multiple of 5

{5, 10, 15, 20, 25, 30, 35, 40, 45, 50}

no. of mutiples of 5 = (10)

$\therefore P(E_2) = \frac{10}{50}$

multiples of both 3 and 5 are

$\{15, 30, 45\}$

no. of multiples of both 3 and 5 are 3

$P(E_1 \cap E_2) = \frac{3}{50}$

Prob that the card is multiple of 3 or multiple of 5 is

$P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$

$= \frac{16}{50} + \frac{10}{50} - \frac{3}{50}$

$= \frac{16+10-3}{50} = \frac{23}{50}$

Problems on Conditional probability

(1) At a soup kitchen a social worker gathers following data. of those visitors 59% are men, 32% are alcoholic and 21% are male alcoholics. what is the probability that a random male visitor is an alcoholic

Sol: Let M = Males
A = Alcoholics

Given 59% are Men

$\Rightarrow$ prob of males $= P(M) = 0.59$

Given 32% are Alcoholics

$\Rightarrow$ prob of an Alcoholic $= P(A) = 0.32$

Given 21% are male Alcoholics

$\Rightarrow P(M \cap A) = 0.21$

Prob that a male visitor is an alcoholic

$$P\left(\frac{A}{M}\right) = ?$$

$$P\left(\frac{A}{M}\right) = \frac{P(M \cap A)}{P(M)} = \frac{0.21}{0.59} = 0.356.$$

(2) During a study of Auto accidents, the Highway safety Council found that 60 percent of all accidents occur at night, 52 percent are alcohol related, and 37 percent occur at night and alcohol related.

(a) what is the probability that an accident was alcohol related, given that it occured at night?

(b) what is the probability that an accident occured at night, given that it was alcohol related?

Sol: Let N = Night
A = Alcohol related

Given that 60% accidents occur at Night

$\Rightarrow$ Probability that an accident occurs at Night $= P(N) = 0.60$

Given that 52% of the accidents are alcohol related.

$\Rightarrow$ Prob that an accident is alcohol related $P(A) = 0.52$

Given 37% occur at Night and alcohol related

$\therefore P(N \cap A) = 37\% = 0.37$

(a) Prob that an accident was alcohol related, given that it occured at Night?

$$P\left(\frac{A}{N}\right) = ?$$

$$P\left(\frac{A}{N}\right) = \frac{P(N \cap A)}{P(N)} = \frac{0.37}{0.60} = 0.62$$

(b) Prob that an accident occured at Night, given that it was alcohol related

$$P\left(\frac{N}{A}\right) = ?$$

$$P\left(\frac{N}{A}\right) = \frac{P(N \cap A)}{P(A)} = \frac{0.37}{0.52} = 0.71$$

(3) Two events A and B are statistically dependent. If $P(A) = 0.39$, $P(B) = 0.21$ and $P(A \text{ or } B) = 0.47$ find the probability that

(a) Neither A nor B will occur

(b) Both A and B will occur

(c) A will occur, given that B has occured

(d) B will occur, given that A has occured

Sol: Given $P(A) = 0.39$ $P(B) = 0.21$

$P(A \cup B) = 0.47$

(a) Neither A nor B will occur $\Rightarrow P(\bar{A} \cap \bar{B}) = ?$

ie $P(\bar{A} \cap \bar{B}) = ?$

(ie) $P(\overline{A \cup B})$

$= 1 - P(A \cup B)$

$= 1 - 0.47.$

$= 0.53$

(b) Both A and B will occur

$P(A \cap B) = ?$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$= 0.39 + 0.21 - 0.47$

$P(A \cap B) = 0.13$

(c) A will occur given that B has occured

$P\left(\frac{A}{B}\right) = ?$

$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} = \frac{0.13}{0.21} = 0.62$

(d) B will occur given that A has occured

$P\left(\frac{B}{A}\right) = ?$

$P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)} = \frac{0.13}{0.39} = 0.33$

④ Given that $P(A) = 3/14$, $P(B) = 1/6$, $P(C) = 1/3$, $P(AC) = 1/7$ and $P(B/C) = 5/21$. Find the following Probabilities $P(A/C)$, $P(C/A)$, $P(BC)$, $P(C/B)$

Sol: Given $P(A) = \frac{3}{14}$

$P(B) = 1/6$

$P(C) = 1/3$

$P(A \cap C) = 1/7$

$P(B/C) = 5/21$

(i) $P(A/C) = \frac{P(A \cap C)}{P(C)} = \frac{1/7}{1/3} = \frac{3}{7}$

(ii) $P\left(\frac{C}{A}\right) = \frac{P(A \cap C)}{P(A)} = \frac{1/7}{3/14} = \frac{1}{7} \times \frac{14}{3} = \frac{2}{3}$

(iii) $P(B \cap C) = ?$

Given $P\left(\frac{B}{C}\right) = 5/21$

we Know $P\left(\frac{B}{C}\right) = \frac{P(B \cap C)}{P(C)}$

$\therefore P(C) \cdot P\left(\frac{B}{C}\right) = P(B \cap C)$

ie $P(B \cap C) = P(C) \cdot P\left(\frac{B}{C}\right)$

$= \frac{1}{3} \times \frac{5}{21}$

$P(B \cap C) = \frac{5}{63}$

(iv) $P\left(\frac{C}{B}\right) = ?$

$$P\left(\frac{C}{B}\right) = \frac{P(B \cap C)}{P(B)} = \frac{5/63}{1/6}$$

$$= \frac{5}{63} \times \frac{6}{1} = \frac{30}{63} = \frac{10}{21}$$

(5) what is the probability that in selecting two cards one at a time from a deck with replacement, the second card is

(a) A face card, given that the first card was red?

(b) An ace, given that the first card was a face card?

(c) A black jack, given that the first card was a red ace?

Sol: Given two cards were in succession ie one after one with replacement.

∴ we know when cards were replaced they will become independent events.

(a) A face card given that the first card was red?

Sol: Total no. of cards = 52

no. of face cards = 16

from

Let F = Face Card

R = Red Card

$P\left(\frac{F}{R}\right) = ?$

$\Rightarrow P\left(\frac{F}{R}\right) = P(F) = \frac{16}{52}$

(b) a black jack given that the first card was red ace?

$P\left(\frac{BJ}{RA}\right) = \frac{2}{52}$

BJ → Black Jack

RA → Red Ace.

(6) A pair of dice is Thrown. If the sum of the numbers on their faces is 6, Find the Probability that the one of the dice shows 2.

Sol: Given two dice were tossed and sum of the numbers on their faces is 6

ie (1,5) (2,4) (3,3) (4,2) (5,1)

Probability that one of the dice shows 2

$n = 2 \Rightarrow (2,4)\ (4,2)$

$n = 5$

$P(E) = 2/5$

(7) What is the probability that a couple's second child will be

(a) a boy given that their first child was a girl.

(b) A girl given that their first child was a girl.

Sol: Let B = Boy
G = Girl.

They are independent events.

(a) $P\left(\frac{B}{G}\right) = P(B) = 1/2$

(b) $P\left(\frac{G_2}{G_1}\right) = P(G_2) = P(G) = 1/2$

Problems on Multiplication Theorem

(1) Three cards are drawn from a pack of cards in succession one after one

(a) Find the probability of getting 3 Hearts if cards are replaced

$$\frac{13}{52} \cdot \frac{13}{52} \cdot \frac{13}{52}$$

(b) Prob of getting 3 King cards if cards are replaced.

$$\frac{4}{52} \cdot \frac{4}{52} \cdot \frac{4}{52}$$

(c) The probability of getting 3 red cards if cards are replaced

$$\frac{26}{52} \cdot \frac{26}{52} \cdot \frac{26}{52}$$

(d) The probability of getting 3 diamond cards if cards are not replaced !

$$\frac{13}{52} \cdot \frac{12}{51} \cdot \frac{11}{50}$$

(e) The probability of getting 3 king cards if cards are not replaced.

$$\frac{4}{52} \cdot \frac{3}{51} \cdot \frac{2}{50}$$

(f) The prob of getting a hearts card, a diamond card and club card in the same order (i) if cards are replaced
(ii) if cards are not replaced

(i) $\frac{13}{52} \cdot \frac{13}{52} \cdot \frac{13}{52}$

(ii) $\frac{13}{52} \cdot \frac{13}{51} \cdot \frac{13}{50}$

(g) The prob of getting king, Queen and Ace in the same order (i) if cards are replaced
(ii) if cards are not replaced,

(i) $\frac{4}{52} \cdot \frac{4}{52} \cdot \frac{4}{52}$

(ii) $\frac{4}{52} \cdot \frac{4}{51} \cdot \frac{4}{50}$

(2) A bag contains 6 green 5 black and 4 white balls. Three balls are drawn in succession ~~one~~ one in a line. Find the probability of getting

1. 1 green, 1 black and 1 white
2. First 2 greens, the third black
3. All different coloured balls
4. All the three are black
5. All the three are green

if (a) balls are replaced
(b) balls are not replaced

Sol: (a) balls are ~~not~~ replaced

(1) Total balls $6+5+4 = 15$

$$\frac{6}{15} \cdot \frac{5}{15} \cdot \frac{4}{15}$$

(2) $$\frac{6}{15} \cdot \frac{6}{15} \cdot \frac{5}{15}$$

(3) $$\frac{6}{15} \cdot \frac{5}{15} \cdot \frac{4}{15}$$

(4) $$\frac{5}{15} \cdot \frac{5}{15} \cdot \frac{5}{15}$$

(5) $$\frac{6}{15} \cdot \frac{6}{15} \cdot \frac{6}{15}$$

(b) balls are not replaced

1. $\frac{6}{15} \frac{5}{14} \cdot \frac{4}{13}$

2. $\frac{6}{15} \cdot \frac{5}{14} \cdot \frac{5}{13}$

3. $\frac{6}{15} \cdot \frac{5}{14} \cdot \frac{4}{13}$

4. $\frac{5}{15} \cdot \frac{4}{14} \cdot \frac{3}{13}$

5. $\frac{6}{15} \cdot \frac{5}{14} \cdot \frac{4}{13}$ 3

كل واحد respective

على الترتيب / حسب التعاقب respectively

(3) April 2004 VIP

A problem in probability is given to Three students. A, B and C and they are asked to solve it independently. Their chances of solving it are 1/5, 2/5 and 3/5 respectively. Find the probability that atleast two will solve the problem. It means that solve the problem

respective

Sol: Problem is given to 3 students A, B, C

Probabilities of their solving the problem are

$P(A) = 1/5$ $\Rightarrow P(\bar{A}) = 1 - P(A)$

$= 1 - 1/5 = \frac{5-1}{5}$

$= \frac{4}{5}$

$1 - \frac{2}{3}$ then shift b and a.b/c

$\Rightarrow P(B) = 2/5$

$P(\bar{B}) = 1 - P(B) = 1 - 2/5$

$= \frac{5-2}{5} = 3/5$

$P(C) = 3/5$

$P(\bar{C}) = 1 - P(C)$

$= 1 - 3/5$

$P(\bar{C}) = \frac{5-3}{5} = \frac{2}{5}$

Probability that the problem is solved by atleast two of them is

$P(A \cap B \cap \bar{C}) + P(A \cap \bar{B} \cap C) + P(\bar{A} \cap B \cap C) + P(A \cap B \cap C)$

$= P(A) \cdot P(B) \cdot P(\bar{C}) + P(A) P(\bar{B}) P(C) + P(\bar{A}) P(B) P(C)$

$+ P(A) P(B) P(C)$

$= \frac{1}{5} \cdot \frac{2}{5} \cdot \frac{2}{5} + \frac{1}{5} \cdot \frac{3}{5} \cdot \frac{3}{5} + \frac{4}{5} \cdot \frac{2}{5} \cdot \frac{3}{5} + \frac{1}{5} \cdot \frac{2}{5} \cdot \frac{3}{5}$

$= \frac{4}{125} + \frac{9}{125} + \frac{24}{125} + \frac{6}{125}$

$= \frac{4+9+24+6}{125} = \frac{43}{125}$

VIP

(4) A problem is given to Three students. The probabilities of their solving the problem are $1/2, 1/3, 1/4$ respectively. Find the probability that the problem is solved.

Sol: Let the problem is given to Three students A, B, C.

Their respective probabilities are

$P(A) = 1/2 \quad P(B) = 1/3 \quad P(C) = 1/4$

$P(A) = 1/2 \Rightarrow P(\bar{A}) = 1 - P(A)$

$= 1 - 1/2$

$= \frac{2-1}{2} = 1/2$

$P(\bar{A}) = 1/2$

$P(B) = 1/3 \Rightarrow P(\bar{B}) = 1 - P(B)$

$= 1 - 1/3$

$= \frac{3-1}{3} = 2/3$

$P(\bar{B}) = 2/3$

$P(C) = 1/4 \Rightarrow P(\bar{C}) = 1 - P(C)$

$= 1 - 1/4$

$= \frac{4-1}{4} = \frac{3}{4}$

$P(\bar{C}) = 3/4$

First calculate that the problem is not solved by any one of them

ie $P(\bar{A} \cap \bar{B} \cap \bar{C}) = P(\bar{A}) \cdot P(\bar{B}) \cdot P(\bar{C})$

$= 1/2 \cdot 2/3 \cdot 3/4 = 1/4$

∴ The probability that the problem is solved is $1 - 1/4 = 3/4$.

Problems on BAYES THEOREM

(1) Given The probabilities of Three events A, B, C, occuring are $P(A) = 0.35$ $P(B) = 0.45$ $P(C) = 0.2$. Assuming that A, B or C has occured, The probabilities of another event, X occuring are $P(\frac{X}{A}) = 0.8$ $P(\frac{X}{B}) = 0.65$ $P(\frac{X}{C}) = 0.3$

Find $P(\frac{A}{X})$, $P(\frac{B}{X})$, $P(\frac{C}{X})$

Sol:

Given $P(A) = 0.35$ $P(\frac{X}{A}) = 0.8$

$P(B) = 0.45$ $P(\frac{X}{B}) = 0.65$

$P(C) = 0.2$ $P(\frac{X}{C}) = 0.3$

$$P(X) = P(A) \cdot P(\frac{X}{A}) + P(B) \cdot P(\frac{X}{B}) + P(C) \cdot P(\frac{X}{C})$$

$$= (0.35)(0.8) + (0.45)(0.65) + (0.2)(0.3)$$

$$= 0.28 + 0.2925 + 0.06$$

$$P(X) = 0.6325$$

$$P(\frac{A}{X}) = \frac{P(A \cap X)}{P(X)} = \frac{P(A) \cdot P(\frac{X}{A})}{P(X)} = \frac{(0.35)(0.8)}{0.6325}$$

$$= \frac{0.28}{0.6325} = 0.443$$

$$P(\frac{B}{X}) = \frac{P(B \cap X)}{P(X)} = \frac{P(B) \cdot P(\frac{X}{B})}{P(X)} = \frac{(0.45)(0.65)}{0.6325}$$

$$= \frac{0.2925}{0.6325} = 0.4625$$

$$P\left(\frac{C}{X}\right) = \frac{P(C \cap X)}{P(X)} = \frac{P(C) \cdot P\left(\frac{X}{C}\right)}{P(X)} = \frac{(0.2)(0.3)}{0.6235}$$

$$= \frac{0.06}{0.6235} = 0.0949.$$

(2) Two related experiments were performed. The first has three possible ~~outcomes~~ mutually exclusive outcomes. A, B and C. The second has two possible mutually exclusive outcomes X and Y. We know $P(A) = 0.2$ and $P(B) = 0.65$. We also know the following conditional probabilities if the result of the second experiment is X; $P\left(\frac{X}{A}\right) = 0.75$ $P\left(\frac{X}{B}\right) = 0.60$ $P\left(\frac{X}{C}\right) = 0.40$. Find $P\left(\frac{A}{X}\right)$, $P\left(\frac{B}{X}\right)$ and $P\left(\frac{C}{X}\right)$. What is the probability that the result of the second experiment is Y?

Sol: Given First experiment has three possible mutually exclusive events A, B, C.

Give $P(A) = 0.2$

$P(B) = 0.65$

$P(A) + P(B) + P(C) = 1$

$\therefore P(C) = 1 - [P(A) + P(B)]$

$= 1 - [0.2 + 0.65]$

$= 1 - 0.85 = 0.15$

ie $P(A) = 0.2$

$P(B) = 0.65$

$P(C) = 0.15$

Given $P\left(\frac{X}{A}\right) = 0.75$

$P\left(\frac{X}{B}\right) = 0.60$

$P\left(\frac{X}{C}\right) = 0.40$

$$P(X) = P(A)\cdot P\left(\frac{X}{A}\right) + P(B)\cdot P\left(\frac{X}{B}\right) + P(C)\cdot P\left(\frac{X}{C}\right)$$

$$= (0.2)(0.75) + (0.65)(0.60) + (0.15)(0.40)$$

$$= 0.15 + 0.39 + 0.06$$

$$\boxed{P(X) = 0.6}$$

$$P\left(\frac{A}{X}\right) = \frac{P(A \cap X)}{P(X)} = \frac{P(A)\cdot P\left(\frac{X}{A}\right)}{P(X)}$$

$$= \frac{(0.2)(0.75)}{0.6}$$

$$= \frac{0.15}{0.6} = 0.25$$

$$P\left(\frac{B}{X}\right) = \frac{P(B \cap X)}{P(X)} = \frac{P(B)\cdot P\left(\frac{X}{B}\right)}{P(X)}$$

$$= \frac{(0.65)(0.60)}{(0.6)}$$

$$= 0.65$$

$$P\left(\frac{C}{X}\right) = \frac{P(C \cap X)}{P(X)} = \frac{P(C)\cdot P\left(\frac{X}{C}\right)}{P(X)}$$

$$= \frac{(0.15)(0.40)}{(0.60)}$$

$$= 0.10$$

Given X and Y are Mutually exclusive

$$\therefore P(X) + P(Y) = 1$$

$$(0.6) + P(Y) = 1$$

$$P(Y) = 1 - 0.60 = 0.40$$

Prob that the result of the second experiment is Y $\Rightarrow P(Y) = 0.40$

April 2004

(3) Assume that a factory has two machines. Past records show that machine-I produces 30% of the output and the machine-II produces the rest. Further, 5% of the items produced by machine-I were defective and only 1% produced by machine-II were defective. If a defective item is drawn at random, what is probability that the defective item produced by machine (a) I (b) II

Sol: Given only two machines produce the out

Give $P(M_1) = 30\%$ rest is Machine-II

$\therefore P(M_2) = (100-30)\% = 0.70$

$P(M_1) = 30\% = 0.30$

$P(M_2) = 70\% = 0.70$

Given 5% items produced machine-I are defective

1% items produced machine-II are defectiv

$\therefore P\left(\frac{D}{M_1}\right) = 5\% = 0.05$

$P\left(\frac{D}{M_2}\right) = 1\% = 0.01$

$P(D) = ?$

$P(D) = P(M_1)\,P\left(\frac{D}{M_1}\right) + P(M_2)\cdot P\left(\frac{D}{M_2}\right)$

$= (0.30)(0.05) + (0.70)(0.01)$

$= 0.015 + 0.007$

$= 0.022$

If a defective item is drawn at random, what is the probability that it was produced by (i) Machine-I

(ii) Machine-II

(i) $P\left(\frac{M_1}{D}\right) = ?$

$$P\left(\frac{M_1}{D}\right) = \frac{P(M_1 \cap D)}{P(D)} = \frac{P(M_1) \cdot P\left(\frac{D}{M_1}\right)}{P(D)}$$

$$= \frac{(0.30)(0.05)}{0.022}$$

$$P\left(\frac{M_1}{D}\right) = \frac{0.015}{0.022} = 0.68$$

(ii) $P\left(\frac{M_2}{D}\right) = \frac{P(M_2 \cap D)}{P(D)} = \frac{P(M_2) \cdot P\left(\frac{D}{M_2}\right)}{P(D)}$

$$= \frac{(0.70)(0.01)}{0.022}$$

$$P\left(\frac{M_2}{D}\right) = \frac{0.007}{0.022} = 0.32$$

(iv) $P(D) = P(M_1) P\left(\frac{D}{M_1}\right) + \cdots$

(iii) $P\left(\frac{M_1}{D}\right) =$

(vi) $P\left(\frac{M_2}{D}\right) =$

(v) $P\left(\frac{M}{D}\right) =$